

Warm inflationary model in loop quantum cosmology

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(Dated: June 8, 2010)

Abstract

A warm inflationary universe model in loop quantum cosmology is studied. In general we discuss the condition of inflation in this framework. By using a chaotic potential, $V(\phi) \propto \phi^2$, we develop a model where the dissipation coefficient $\Gamma = \Gamma_0 = \text{constant}$. We use recent astronomical observations for constraining the parameters appearing in our model.

PACS numbers: 98.80.Cq

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I. INTRODUCTION

It is well known that warm inflation, as opposed to the conventional cool inflation, presents the attractive feature that it avoids the reheating period [1]. In these kind of models dissipative effects are important during the inflationary period, so that radiation production occurs concurrently together with the inflationary expansion. If the radiation field is in a highly excited state during inflation, and this has a strong damping effect on the inflaton dynamics, then it is found a strong regimen of warm inflation. Also, the dissipating effect arises from a friction term which describes the processes of the scalar field dissipating into a thermal bath via its interaction with other fields. Warm inflation shows how thermal fluctuations during inflation may play a dominant role in producing the initial fluctuations necessary for large-scale structure formation. In these kind of models the density fluctuations arise from thermal rather than quantum fluctuations [2]. These fluctuations have their origin in the hot radiation and influence the inflaton through a friction term in the equation of motion of the inflaton scalar field [3]. Among the most attractive features of these models, warm inflation ends when the universe heats up to become radiation domination; at this epoch the universe stops inflating and "smoothly" enters in a radiation dominated Big-Bang phase [4, 5]. The matter components of the universe are created by the decay of either the remaining inflationary field or the dominant radiation field [6].

On the other hand, Loop Quantum Gravity (LQG) is a resulting nonperturbative background independent approach to quantize gravity [7]. Here, the geometry in LQG is discrete and the continuum space-time is obtained from quantum geometry in a large eigenvalue limit. The application of LQG techniques to homogeneous space-times results in LQC which has directed to important insights on the resolution of singularities [8–11]. Within the various conceivable cosmological models the ones which are best understood are the Friedmann-Robertson-Walker (FRW) models [12]. In this case it has been shown that the quantum isotropic and homogeneous gravitational degrees of freedom minimally coupled to the massless scalar field allow non-singular evolution for the open, closed and flat universes. Here, the singularity becomes substituted with the smooth Big Bounce. In this sense the initial singularity is resolved by the quantum gravitational repulsion effects. Because of the loop quantum effect the standard Friedmann equation can be modified by adding a correction term ρ^2 at the scale when ρ becomes comparable to a critical density $\rho_c \approx 0.82 G^{-2}$ (G is the

Newton's gravitational constant) which is close to the Planck density. Within the framework of LQC the inflationary model has been considered in Ref.[13]. Recently, the dynamics of the interacting dark energy model in Einstein and loop quantum cosmology was considered in [14], and the cosmological evolution of the interacting phantom (quintessence) model in loop quantum gravity was studied in Ref.[15].

The main goal of the present work is to investigate the possible realization of a warm inflationary universe model, within the framework of the effective theory of loop quantum cosmology. In this way, we study warm-LQC model and the cosmological perturbations, which are expressed in term of different parameters appearing in our model. These parameters are constrained from the WMAP 5-year data [16]. Also, we only discuss the normal inflation epoch, i.e., after the super-inflation scenario. For a review of super-inflation, see, e.g., [13, 17].

The outline of the paper is as follows. The next section presents a short review of the effective theory of LQC. In Section III we present the warm inflationary phase in this framework. Section IV deals with the scalar and tensor perturbations, respectively. In Section V we use a chaotic potential and $\Gamma = \Gamma_0 = \text{constant}$, for obtaining explicit expression for our model. Finally, Sec.VI summarizes our findings. We chose units so that $c = \hbar = 1$.

II. LOOP QUANTUM COSMOLOGY

The effective Friedmann equation can be obtained by using an effective Hamiltonian with loop quantum modifications [18–20]

$$\mathcal{H}_{eff} = -\frac{3}{\kappa \gamma^2 \bar{\mu}^2} a \sin^2(\bar{\mu} \mathbf{c}) + \mathcal{H}_M, \quad (1)$$

where $\kappa = 8\pi G$, γ is the dimensionless Barbero-Immirzi parameter ($\gamma \approx 0.2375$ see Ref.[21]), $\bar{\mu}$ is inferred as the kinematical length of the square loop, and \mathcal{H}_M is the matter Hamiltonian. Here, \mathbf{c} and \mathbf{p} are, respectively, conjugate connection and triad satisfying $\{\mathbf{c}, \mathbf{p}\} = \gamma\kappa/3$, and the relation with the metric components of the FRW becomes

$$\mathbf{c} = \gamma \dot{a}, \quad \text{and} \quad \mathbf{p} = a^2, \quad (2)$$

where a represents the scale factor. The modified Friedmann equation can be found by using Hamiltons equations for $\dot{\mathbf{p}}$,

$$\dot{\mathbf{p}} = \{\mathbf{p}, \mathcal{H}_{eff}\} = \frac{2a}{\gamma \bar{\mu}} \sin(\bar{\mu} \mathbf{c}) \cos(\bar{\mu} \mathbf{c}), \quad (3)$$

and from Eq.(2) implies that \dot{a} is given by

$$\dot{a} = \frac{1}{\gamma \bar{\mu}} \sin(\bar{\mu} \mathbf{c}) \cos(\bar{\mu} \mathbf{c}). \quad (4)$$

Furthermore, the vanishing of the Hamiltonian constraint implies[18–20]

$$\sin^2(\bar{\mu} \mathbf{c}) = \frac{\kappa \gamma^2 \bar{\mu}^2}{3 a} \mathcal{H}_M. \quad (5)$$

From Eqs.(4) and (5), the effective Friedmann equation becomes

$$H^2 = \frac{\kappa}{3} \rho \left[1 - \frac{\rho}{\rho_c} \right], \quad (6)$$

where $H = \dot{a}/a$ is the Hubble parameter, $\rho_c = \sqrt{3} \rho_p / (16\pi^2 \gamma^3)$ is the critical loop quantum density and ρ_p is the Planck density equal to $\rho_p = G^{-2}$.

In the following we will consider a total energy density $\rho = \rho_\phi + \rho_\gamma$ where ϕ corresponds to a self-interacting scalar field with energy density, ρ_ϕ , given by $\rho_\phi = \frac{1}{2} \dot{\phi}^2 + V(\phi)$, and ρ_γ represents the radiation energy density.

III. WARM-LQC INFLATIONARY PHASE

The dynamics of the cosmological model in the warm-LQC inflationary scenario is described by the equations

$$\ddot{\phi} + 3H \dot{\phi} + V' = -\Gamma \dot{\phi}, \quad (7)$$

and

$$\dot{\rho}_\gamma + 4H\rho_\gamma = \Gamma \dot{\phi}^2. \quad (8)$$

Here Γ is the dissipation coefficient and it is responsible for the decay of the scalar field into radiation during the inflationary era. In general, Γ can be assumed to be a constant or a function of the scalar field ϕ , or the temperature of the thermal bath T_r , or both[1, 22–24]. On the other hand, Γ must satisfy $\Gamma > 0$ by the Second Law of Thermodynamics.

Statistical mechanics of quantum open systems has shown that the interaction of quantum field with a thermal bath can be characterized by a fluctuation dissipation relation[25]. These

effects support the idea of introducing a friction term into the field equation of motion. Here, the friction term $\Gamma\dot{\phi}$ describes the interaction between the scalar field ϕ and the heat bath. The possibility of warm inflation arising in realistic particles models has been enhanced by the decay mechanism in supersymmetric theories, where the inflaton decays into radiation fields as a consequence of the heavy particle intermediate[26]. If the coupling constants are sufficiently large, these models can lead to warm inflation[27]. Dots mean derivatives with respect to time and $V' = \partial V(\phi)/\partial\phi$.

During the inflationary epoch the energy density associated to the scalar field is of the order of the potential, i.e. $\rho_\phi \sim V$, and dominates over the energy density associated to the radiation field, i.e. $\rho_\phi > \rho_\gamma$. Assuming the set of slow-roll conditions, i.e. $\dot{\phi}^2 \ll V(\phi)$, and $\ddot{\phi} \ll (3H + \Gamma)\dot{\phi}$ [1], the Friedmann equation (6) reduces to

$$H^2 \approx \frac{\kappa}{3} V \left[1 - \frac{V}{\rho_c} \right], \quad (9)$$

and Eq. (7) becomes

$$3H[1 + R]\dot{\phi} \approx -V', \quad (10)$$

where R is the rate defined as

$$R = \frac{\Gamma}{3H}. \quad (11)$$

For the strong (weak) dissipation regime, we have $R \gg 1$ ($R < 1$).

We also consider that during warm inflation the radiation production is quasi-stable, i.e. $\dot{\rho}_\gamma \ll 4H\rho_\gamma$ and $\dot{\rho}_\gamma \ll \Gamma\dot{\phi}^2$. From Eq.(8) we obtained that the energy density of the radiation field becomes

$$\rho_\gamma = \frac{\Gamma\dot{\phi}^2}{4H}, \quad (12)$$

which could be written as $\rho_\gamma = \sigma T_r^4$, where σ is the Stefan-Boltzmann constant and T_r is the temperature of the thermal bath. By using Eqs.(10), (11) and (12) we get

$$\rho_\gamma = \frac{RV'^2}{4\kappa V(1 - V/\rho_c)(1 + R)^2}. \quad (13)$$

Introducing the dimensionless slow-roll parameter, we write

$$\varepsilon \equiv -\frac{\dot{H}}{H^2} \simeq \frac{V'^2}{2\kappa(1 + R)V^2} \left[\frac{(1 - 2V/\rho_c)}{(1 - V/\rho_c)^2} \right], \quad (14)$$

and the second slow-roll parameter η becomes

$$\eta \equiv -\frac{\ddot{H}}{H\dot{H}} \simeq \frac{1}{\kappa V(1 - V/\rho_c)(1 + R)} \left[V'' - \frac{2V'^2}{\rho_c(1 - 2V/\rho_c)} - \frac{V'^2(1 - 2V/\rho_c)}{2V(1 - V/\rho_c)} \right]. \quad (15)$$

We see that for $R = 0$ (or $\Gamma = 0$), the parameters ε and η given by Eqs.(14) and (15) respectively, are reduced to the typical expression for cool inflation in LQC[13]. Note that the term in the bracket of Eq.(14) is the correction to the standard warm inflationary model.

It is possible to find a relation between the energy densities ρ_γ and ρ_ϕ given by

$$\rho_\gamma = \frac{R}{2(1+R)} \left[\frac{(1 - \rho_\phi/\rho_c)}{(1 - 2\rho_\phi/\rho_c)} \right] \varepsilon \rho_\phi \simeq \frac{R}{2(1+R)} \left[\frac{(1 - V/\rho_c)}{(1 - 2V/\rho_c)} \right] \varepsilon V. \quad (16)$$

Recall that during inflation the energy density of the scalar field becomes dominated by the potential energy, i.e. $\rho_\phi \sim V$.

The condition which the warm inflation epoch on a LQC could take place can be summarized with the parameter ε satisfying the inequality $\varepsilon < 1$. This condition is analogue to the requirement that $\ddot{a} > 0$. The condition given above is rewritten in terms of the densities by using ρ_γ , we get

$$\left[\frac{(1 - \rho_\phi/\rho_c)}{(1 - 2\rho_\phi/\rho_c)} \right] \rho_\phi > \frac{2(1+R)}{R} \rho_\gamma. \quad (17)$$

Inflation ends when the universe heats up at a time when $\varepsilon \simeq 1$, which implies

$$\left[\frac{V_f'}{V_f} \right]^2 \left[\frac{(1 - 2V_f/\rho_c)}{\kappa(1 - V_f/\rho_c)^2} \right] \simeq 2(1 + R_f). \quad (18)$$

The number of e-folds at the end of inflation is given by

$$N \simeq -\kappa \int_{\phi_*}^{\phi_f} \frac{V}{V'} (1 - V/\rho_c) (1 + R) d\phi'. \quad (19)$$

In the following, the subscripts $*$ and f are used to denote to the epoch when the cosmological scales exit the horizon and the end of inflation, respectively.

IV. PERTURBATIONS

In this section we will study the scalar and tensor perturbations for our model. Note that in the case of scalar perturbations the scalar and the radiation fields are interacting. Therefore, isocurvature (or entropy) perturbations are generated besides the adiabatic ones. This occurs because warm inflation can be considered as an inflationary model with two basic fields [28, 29]. In this context dissipative effects can produce a variety of spectral, ranging between red and blue [2, 28], and thus producing the running blue to red spectral suggested by WMAP five-year data[16].

As argued in Ref.[13] for LQC, the density perturbation could be written as $\delta_H = \frac{2}{5} \frac{H}{\phi} \delta\phi$ [30]. From Eqs.(10) and (11), the latter equation becomes

$$\delta_H^2 = \frac{36}{25} \frac{H^4 (1+R)^2}{V'^2} \delta\phi^2 = \frac{4}{25} \left(\frac{\kappa^2 V^2 (1-V/\rho_c)^2 (1+R)^2}{V'^2} \right) \delta\phi^2. \quad (20)$$

The scalar field presents fluctuations which are due to the interaction between the scalar and the radiation fields. In the case of strong dissipation, the dissipation coefficient Γ is much greater than the rate expansion H , i.e. $R = \Gamma/3H \gg 1$ and following Taylor and Berera[31], we can write

$$(\delta\phi)^2 \simeq \frac{k_F T_r}{2\pi^2}, \quad (21)$$

where the wave-number k_F is defined by $k_F = \sqrt{\Gamma H/V} = H \sqrt{3R} \geq H$, and corresponds to the freeze-out scale at which dissipation damps out to the thermally excited fluctuations. The freeze-out wave-number k_F is defined at the point where the inequality $V_{,\phi\phi} < \Gamma H$, is satisfied [31, 32].

From Eqs. (20) and (21) it follows that

$$\delta_H^2 \approx \frac{2}{25\pi^2} \left[\frac{T_r}{V'^2} \right] [\kappa R V (1-V/\rho_c)]^{5/2}. \quad (22)$$

The scalar spectral index n_s is given by $n_s - 1 = \frac{d \ln \delta_H^2}{d \ln k}$, where the interval in wave number is related to the number of e-folds by the relation $d \ln k(\phi) = -dN(\phi)$. From Eq.(22), we get

$$n_s \approx 1 - [5\tilde{\varepsilon} - 2\tilde{\eta} - \zeta], \quad (23)$$

where, the slow-roll parameters $\tilde{\varepsilon}$, $\tilde{\eta}$ and ζ , (for $R \gg 1$) are given by

$$\tilde{\varepsilon} \approx \frac{1}{2\kappa R} \left[\frac{V'}{V} \right]^2 \frac{(1-2V/\rho_c)}{(1-V/\rho_c)^2}, \quad (24)$$

$$\tilde{\eta} \approx \frac{1}{\kappa V (1-V/\rho_c) R} \left[V'' - \frac{2V'^2}{\rho_c (1-2V/\rho_c)} - \frac{V'^2 (1-2V/\rho_c)}{2V (1-V/\rho_c)} \right],$$

and

$$\zeta \approx \frac{V'^2}{\kappa V (1-V/\rho_c) R} \left[\frac{4}{\rho_c (1-2V/\rho_c)} + \frac{(1-2V/\rho_c)}{V (1-V/\rho_c)} \right] - \frac{5}{2} \frac{V'}{\kappa V (1-V/\rho_c)} \frac{R'}{R^2},$$

respectively.

One of the interesting features of the five-year data set from WMAP is that it hints at a significant running in the scalar spectral index $dn_s/d \ln k = \alpha_s$ [16]. From Eq.(23) we obtain that the running of the scalar spectral index becomes

$$\alpha_s \approx 2\tilde{\varepsilon} \frac{V(1-V/\rho_c)}{V'(1-2V/\rho_c)} [5\tilde{\varepsilon}' - 2\tilde{\eta}' - \zeta']. \quad (25)$$

In models with only scalar fluctuations the marginalized value for the derivative of the spectral index is approximately -0.05 from WMAP-five year data only [16].

Tensor perturbation do not couple strongly to the thermal background and so gravitational waves are only generated by quantum fluctuations (as in standard inflation) [31]. The corresponding spectrum becomes

$$A_g^2 = 8\kappa \left(\frac{H}{2\pi} \right)^2 = \frac{2\kappa^2}{3\pi^2} V(1-V/\rho_c). \quad (26)$$

For $R \gg 1$ and from expressions (22) and (26) we may write the tensor-scalar ratio as

$$r(k) = \left(\frac{A_g^2}{P_{\mathcal{R}}} \right) \Big|_{k_*} \simeq \frac{4}{3\kappa^{1/2}} \left[\frac{V'^2}{T_r V^{3/2} (1-V/\rho_c)^{3/2} R^{5/2}} \right] \Big|_{k=k_*}. \quad (27)$$

Here, $\delta_H \equiv 2P_{\mathcal{R}}^{1/2}/5$ and k_* is referred to $k = Ha$, the value when the universe scale crosses the Hubble horizon during inflation.

Combining WMAP observations [16] with the Sloan Digital Sky Survey (SDSS) large scale structure surveys [33], it is found an upper bound for r given by $r(k_* \simeq 0.002 \text{ Mpc}^{-1}) < 0.28$ (95% C.L.), where $k_* \simeq 0.002 \text{ Mpc}^{-1}$ corresponds to $l = \tau_0 k \simeq 30$, with the distance to the decoupling surface $\tau_0 = 14,400 \text{ Mpc}$. The SDSS measures galaxy distributions at redshifts $a \sim 0.1$ and probes k in the range $0.016 h \text{ Mpc}^{-1} < k < 0.011 h \text{ Mpc}^{-1}$. The recent WMAP observation results give the values for the scalar curvature spectrum $P_{\mathcal{R}}(k_*) \equiv 25\delta_H^2(k_*)/4 \simeq 2.3 \times 10^{-9}$ and the scalar-tensor ratio $r(k_*) < 0.2$.

From Eqs.(22) and (24), we can write

$$V^{1/2} (1-\nu)^{1/2} (1-\nu) = \frac{4\pi^2 P_{\mathcal{R}}}{T_r (\kappa R)^{3/2}} \tilde{\varepsilon}, \quad (28)$$

where $\nu = \nu(\phi)$ is defined by

$$\nu = \frac{V(\phi)}{\rho_c}.$$

Here, ν describe the quantum geometry effects in LQC, and is a small quantity $\nu < 10^{-9}$ (see Ref.[13]). The approximate value of the critical density ρ_c in the effective theory of LQC is $\rho_c \approx 0.82 \rho_p$, where the Planck density $\rho_p = G^{-2} = m_p^4$, so we have $\rho_c \approx 0.82 m_p^4$. By using the WMAP observations where $P_{\mathcal{R}} \simeq 2.3 \times 10^{-9}$, and in view of $\nu \ll 1$, we get

$$\nu \simeq \frac{16\pi^4 P_{\mathcal{R}}^2}{\rho_c T_r^2 \kappa^3 R^3} \tilde{\varepsilon}^2 \simeq 6.3 \times 10^{-19} \frac{m_p^2}{T_r^2 R^3} \tilde{\varepsilon}^2. \quad (29)$$

In the case of strong dissipation $R \gg 1$, then we find from Eq.(29) an upper limit for ν , and it becomes

$$\nu \ll 10^{-18} \left[\frac{m_p}{T_r} \right]^2 \tilde{\varepsilon}^2. \quad (30)$$

Note that this inequality for ν become dependent of the temperature of the thermal bath T_r and $\tilde{\varepsilon}^2$. If we compared with respect to standard supercooled inflation, $\nu \simeq 10^{-9} \tilde{\varepsilon}$ [13]. Note also, that this upper limit for ν increase when the temperature T_r decreases.

V. AN EXAMPLE: CHAOTIC POTENTIAL IN THE STRONG DISSIPATION APPROACH

Let us consider an inflaton scalar field ϕ with a chaotic potential. We write for the chaotic potential as $V = m^2 \phi^2 / 2$, where m is the mass of the scalar field. An estimation of this parameter is given for LQC in Ref.[13] and for warm inflation in Ref.[31]. In the following, we develop the model for a constant dissipation coefficient $\Gamma = \Gamma_0 = \text{const.}$, and we will restrict ourselves to the strong dissipation regime, i.e. $R \gg 1$.

By using the chaotic potential, we find that from Eq.(10)

$$\dot{\phi} = -\frac{m^2 \phi}{\Gamma_0} \implies \phi(t) = \phi_0 e^{-m^2 t / \Gamma_0}, \quad (31)$$

and during the inflationary scenario the scalar field decays due to dissipation into the radiation field. The Hubble parameter is given by

$$H(t) = \frac{m \kappa^{1/2} \phi_0}{\sqrt{6}} e^{-m^2 t / \Gamma_0} \left[1 - \frac{m^2 \phi_0^2}{2 \rho_c} e^{-2m^2 t / \Gamma_0} \right]^{1/2}. \quad (32)$$

Note that in the limit $\rho_c \gg \rho_\phi \simeq V$ the Hubble parameter coincide with Ref.[31]. The dissipation parameter R in this case is

$$R(t) = \frac{\sqrt{2} \Gamma_0}{\sqrt{3} \kappa m \phi_0} e^{m^2 t / \Gamma_0} \left[1 - \frac{m^2 \phi_0^2}{2 \rho_c} e^{-2m^2 t / \Gamma_0} \right]^{-1/2}.$$

By integrating Eq.(19) the number of e-folds results in

$$N = -\frac{\Gamma_0 \sqrt{\rho_c \kappa}}{2\sqrt{3} m^2} [h(\nu_f) - h(\nu_*)], \quad (33)$$

where $h(\nu) = \arcsin(\sqrt{\nu}) + \sqrt{\nu(1-\nu)}$. From the condition that $\tilde{\varepsilon} \simeq 1$ (see Eq.(18)) at the end of warm-LQC inflation, we find that the magnitude of ν at this time is $\nu_f \approx \frac{3m^4}{\kappa \rho_c \Gamma_0^2}$.

By using Eq.(16), we can relate the energy density of the radiation field to the energy density of the inflaton field to

$$\rho_\gamma = \left(\frac{\sqrt{3} m^2}{2 \Gamma_0} \right) \left[\frac{\rho_\phi}{\kappa (1 - \rho_\phi / \rho_c)} \right]^{1/2}. \quad (34)$$

Note again that in the limit $\rho_c \gg \rho_\phi$, Eq.(34) coincides with that corresponding to the case where LQC is absent[31], i.e., $\rho_\gamma \propto \rho_\phi^{1/2}$.

From Eq.(22), we obtain that the scalar power spectrum becomes

$$P_{\mathcal{R}}(k) \approx \left(\frac{T_r}{4 \pi^2 m^2} \right) \left(\frac{\Gamma_0}{\sqrt{3}} \right)^{5/2} \left(\kappa^{5/4} V^{1/4} [1 - \nu]^{5/4} \right) \Big|_{k=k_*}, \quad (35)$$

and from Eq.(27) the tensor-scalar ratio is given by

$$r(k) \approx \left(\frac{3^{1/4} 8 \kappa^{3/4} m^2}{T_r \Gamma_0^{5/2}} \right) \left[\frac{V^3}{(1 - \nu)} \right]^{1/4} \Big|_{k=k_*}. \quad (36)$$

By using the WMAP observations where $P_{\mathcal{R}}(k_*) \simeq 2.3 \times 10^{-9}$, $r(k_*) < 0.2$ and $\nu \ll 1$, we obtained from Eqs.(35) and (36) that

$$\nu_* = \frac{V_*}{\rho_c} < 4.2 \times 10^{-12}. \quad (37)$$

From Eqs.(35) and (37), we get the inequality

$$m^2 < 5 \times 10^4 \frac{T_r \Gamma_0^{5/2}}{m_p^{3/2}}. \quad (38)$$

Now we consider the special case in which we fix $T_r \simeq 0.24 \times 10^{16}$ GeV, and $\Gamma_0 \simeq 0.5 \times 10^{13}$ GeV (see Ref.[34]). In this special case we obtained that the upper limit for the square mass of the scalar field, is given by $m^2 < 5 \times 10^{-14} m_p^2$.

In Fig.(1) we show the dependence of the tensor-scalar ratio r on the spectral index n_s , for the chaotic model $V = m^2 \phi^2 / 2$. From left to right $m = 10^{-7} m_p$ (dashed line) and $m = 10^{-8} m_p$ (solid line), respectively. From Ref.[16], two-dimensional marginalized constraints (68% and 95% confidence levels) on inflationary parameters r and n_s , the spectral index of fluctuations, defined at $k_0 = 0.002$ Mpc⁻¹. The five-year WMAP data places stronger limits on r (shown in blue) than three-year data (grey)[35]. In order to write down values that relate n_s and r , we used Eqs.(23) and (36). Also we have used the values $T_r \simeq 0.24 \times 10^{16}$ GeV, $\Gamma_0 \simeq 0.5 \times 10^{13}$ GeV, and $\rho_c \approx 0.82 m_p^4$.

From Eqs.(33) and (36) , we observed that for $m = 10^{-7} m_p$, the curve $r = r(n_s)$ (see Fig. (1)) for WMAP 5-years enters the 95% confidence region where the ratio $r \simeq 0.42$, which corresponds to the number of e-folds, $N \simeq 66.5$. For $m = 10^{-8} m_p$, $r \simeq 0.44$ corresponds to $N \simeq 694$. From the 68% confidence region for $m = 10^{-7} m_p$, $r \simeq 0.28$, which corresponds to $N \simeq 50.2$. For $m = 10^{-8} m_p$, $r \simeq 0.29$ corresponds to $N \simeq 539$.

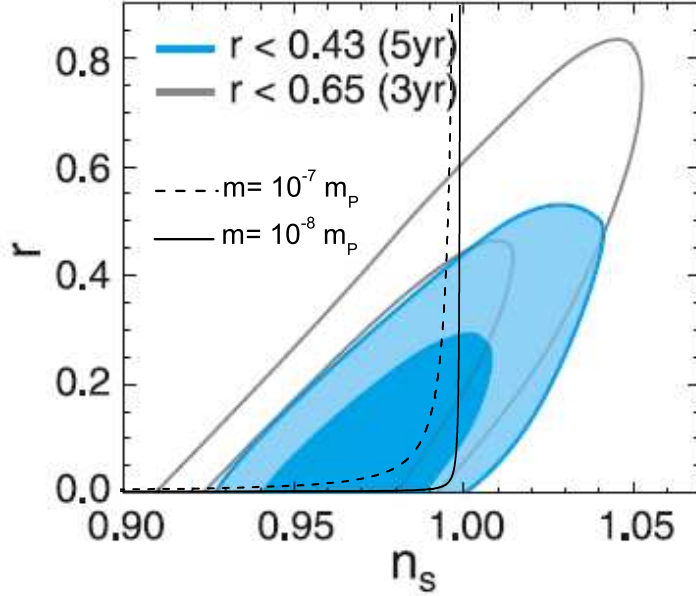


FIG. 1: The plot shows r versus n_s for two values of m . Here, we have fixed the values $T_r \simeq 0.24 \times 10^{16}$ GeV, $\Gamma_0 \simeq 0.5 \times 10^{13}$ GeV, and $\rho_c \approx 0.82 m_p^4$, respectively. The five-year WMAP data places stronger limits on the tensor-scalar ratio (shown in blue) than three-year data (grey) [35].

VI. CONCLUSIONS

In this paper we have investigated the warm inflationary scenario in LQC. In the slow-roll approximation we have found a general relationship between the radiation and scalar field energy densities. This has led us to a general criterium for warm inflation in LQC to occur (see Eq.(17)).

Our specific model is described by a chaotic potential and we have considered the case

in which the dissipation coefficient, $\Gamma = \Gamma_0 = \text{constant}$. Here, we have found that the condition for ν_* presents the same characteristic that occurs in cool inflation for the LQC [13], except that it depends on the extra parameter T_r . In this case, we have obtained the explicit expressions for the corresponding scalar spectrum index and the running of the scalar spectrum index. We also demonstrated that the scalar spectral index, its running and the tensor-to-scalar ratio can be expressed in terms of slow-roll parameters as well as the LQC parameter ν .

In order to bring some explicit results we have taken the constraint in the $r - n_s$ plane to the chaotic model, $V = m^2 \phi^2/2$. We noted that the parameter m , which is bounded from below, $m^2 < 5 \times 10^{-14} m_p^2$, (see Eq.(38)) and the model is well supported by the data as could be seen from Fig.(1). Here, we have used the values $T_r \simeq 0.24 \times 10^{16}$ GeV, $\Gamma_0 \simeq 0.5 \times 10^{13}$ GeV, and $\rho_c \approx 0.82 m_p^4$, respectively. On the other hand, by using the WMAP observations where $P_{\mathcal{R}}(k_*) \simeq 2.3 \times 10^{-9}$, $r(k_*) < 0.2$ and $\nu \ll 1$, we obtained from Eqs.(35) and (36) that $\nu_* < 4 \times 10^{-12}$. We should note that this inequality for ν_* , becomes small by three order of magnitude when it is compared with the case of standard-LQC[13].

We should note that other properties of this model deserve further study. For example, we have not addressed the non-Gaussian effects during warm inflation (see e.g., Refs.[36–38]). A possible calculation from the non-linearity parameter f_{NL} , would give new constrains on the parameters of the model. Also, a sophisticated analysis would give new constraints on the dissipative coefficient $\Gamma = \Gamma(\phi, T_r)$, the cosmological perturbations in LQC[39], and for warm inflation, see, e.g.,[40]. We hope to return to this point in the near future.

Acknowledgments

This work was supported by COMISION NACIONAL DE CIENCIAS Y TECNOLOGIA through FONDECYT Grant N^o. 1090613, also from PUCV DI-PUCV 2009.

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